A Method for Measuring the Directivity of Directional Couplers*

G. E. SCHAFER† AND R. W. BEATTY†

Summary—This method of measuring directivity requires the measurement of the ratio of powers delivered to the side arm when the normal input arm is connected alternately to an adjustable sliding termination and a sliding short circuit. The short circuit is phased to yield maximum and minimum responses and the amplitudes are averaged. Two techniques of adjusting the termination may be used. One procedure requires zero reflection from the termination. The other procedure requires adjustment for a null at the detector and then measurement of the maximum response due to changing the phase of the termination. The inherent errors of the method are analyzed and found to be within the limits—0.01 to 0.00 db in a specific example.

Introduction

A METHOD to measure directivity of a directional coupler is described. The errors in the method are evaluated and graphs are presented for estimating the total error.

Previously described methods[1,2] required measurement of the combined attenuation of coupling and directivity, the impedance of an auxiliary component, the reversal of the directional coupler, or a combination of these. This method permits measurement of the directivity values up to the entire dynamic range of the attenuation measurement system and completes a measurement by attaching first a short circuit and then an adjustable sliding termination to the same terminal.

Procedure

The arrangement of equipment is indicated in Fig. 1, with the coupler oriented as in Fig. 2. Preliminary adjustments[3] are made to the tuners shown in Fig. 1 so that: 1) the calibrated attenuator is operated in a matched system, (the condition under which it was calibrated) and 2) the reflection coefficient Γ_{2i} (measured at terminal surface 2 of the directional coupler with the normal signal source inactive) has a magnitude less than 0.01.

Procedure 1) is to attach the short circuit and obtain a maximum and minimum amplitude reading by

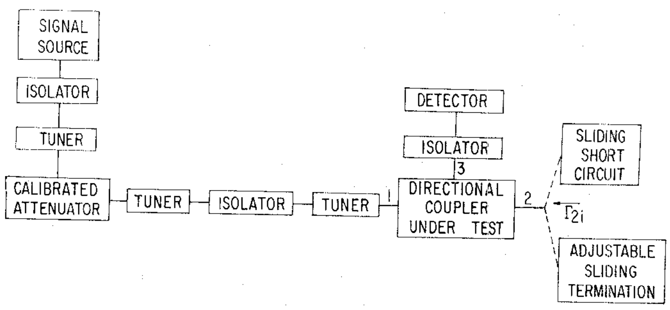

Fig. 1—Arrangement of equipment.

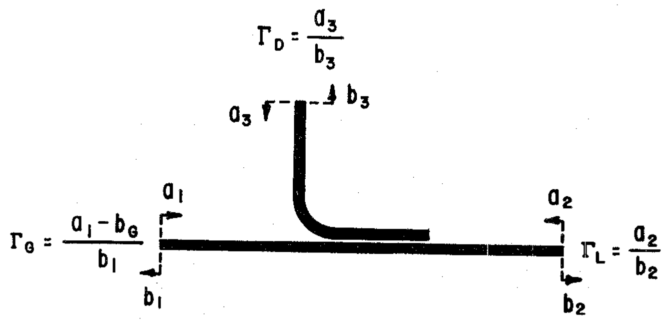

Fig. 2—Representation of directional coupler as a 3-arm junction.

adjusting the phase of the short circuit. The average of these two readings is a reference level and is denoted by $|b_3|_{av}$. An adjustable sliding termination (abbreviated AST in the remainder of the paper) is then attached in place of the sliding short. This AST must be capable of providing zero reflection[4] at the frequency being used. The condition of zero reflection is indicated by no fluctuation in the output level of the detector as the termination is moved along the waveguide. This level is called $|b_3|_0$ and is determined either by the calibrated attenuator or the detector.

$|b_3|_{av}$ is approximately equal to the amplitude of the forward coupled wave, while $|b_3|_0$ is the amplitude of the reverse coupled wave. Therefore, the directivity can be determined to a good approximation by the expression

$$D \approx 20 \log_{10} \frac{|b_3|_{av}}{|b_3|_0}.$$

The adjustment of the AST for $\Gamma_L = 0$ [in procedure 1)] is often tedious. Procedure 2) eliminates this opera-

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† Natl. Bureau of Standards, Boulder, Colo.

[1] C. G. Montgomery, ed., "Techniques of Microwave Measurements," M.I.T. Rad. Lab. Ser., McGraw-Hill Book Co., Inc., New York, N. Y., ch. 14; 1947.

[2] M. Wind and H. Rapaport, "Handbook of Microwave Measurements," Polytechnic Institute of Brooklyn, Microwave Res. Inst., Brooklyn, N. Y., 2nd ed.; 1955.

[3] If the detecting system has sufficient gain, the tuners may be replaced by well-matched broad-band pads with little loss in accuracy. This would simplify the procedure, especially in the case where measurements were to be made at a number of different frequencies.

[4] R. W. Beatty, "An adjustable sliding termination for rectangular waveguide," IRE Trans. on Microwave Theory and Techniques, vol. MTT-5, pp. 192–194; July, 1957.

tion and substitutes instead a null adjustment which is readily obtained.[5] (The short circuit measurement remains unchanged from the first procedure and the significance of $|b_3|_{av}$ is the same.) The AST is manipulated to yield a null in the output of arm 3, and is then moved along the waveguide until a maximum response, $|b_3|_T$, occurs. This is approximately twice $|b_3|_0$. The ratio

$$\frac{|b_3|_{av}}{|b_3|_T}$$

can also be used to determine the directivity to a good approximation by the expression

$$D \approx 20 \log_{10} \frac{2|b_3|_{av}}{|b_3|_T}.$$

Theory

Analysis of this method is accomplished by writing a matrix equation for the three-arm junction representation of a directional coupler

$$b = Sa, \tag{1}$$

(where S is the scattering matrix, and b and a refer respectively to outgoing and incoming wave amplitudes), and solving for the above ratios. The elements of a and b in (1) are displayed in Fig. 2.

The substitution of the appropriate elements into matrix (1) yields a solution for the amplitude of the emergent wave from arm 3,

$$b_3 = -b_g \frac{\begin{vmatrix} S_{12} & 1 - S_{22}\Gamma_L \\ S_{13} & -S_{23}\Gamma_L \end{vmatrix}}{\begin{vmatrix} 1 - S_{11}\Gamma_G & -S_{12}\Gamma_L & -S_{13}\Gamma_D \\ -S_{12}\Gamma_G & 1 - S_{22}\Gamma_L & -S_{23}\Gamma_D \\ -S_{13}\Gamma_G & -S_{23}\Gamma_L & 1 - S_{33}\Gamma_D \end{vmatrix}}, \tag{2}$$

where b_g is the fixed wave amplitude characteristic of the equivalent generator as designated in Fig. 2, and where reciprocity in the form $S_{ij} = S_{ji}$ has been assumed. This may be written in a convenient partition suggested by the form derived by MacPherson and Kerns,[6]

$$|b_3| = \frac{\left| b_g \frac{k}{2} \right|}{\left| \frac{y}{2} + R + r \exp(j\phi) \right|}, \tag{3}$$

where

$$R = \frac{1}{1 - |K\Gamma_L|^2}, \qquad r = |K\Gamma_L| R,$$

$$y = \frac{2\Delta}{KM_{22} - \Delta}, \qquad K = \frac{\begin{vmatrix} S_{12} & S_{22} \\ S_{13} & S_{23} \end{vmatrix}}{S_{13}},$$

$$k = \frac{2S_{13}K}{KM_{22} - \Delta}, \qquad M_{22} = \begin{vmatrix} 1 - S_{11}\Gamma_G & -S_{13}\Gamma_D \\ -S_{13}\Gamma_G & 1 - S_{33}\Gamma_D \end{vmatrix},$$

and

$$\Delta = \begin{vmatrix} 1 - S_{11}\Gamma_G & -S_{12} & -S_{13}\Gamma_D \\ -S_{12}\Gamma_G & -S_{22} & -S_{23}\Gamma_D \\ -S_{13}\Gamma_G & -S_{23} & 1 - S_{33}\Gamma_D \end{vmatrix}.$$

For the arrangement with the short circuit attached, maximum and minimum responses are obtained as the phase of Γ_L is shifted. These responses are given by

$$|b_3| \max = \frac{\left| b_g \frac{k}{2} \right|}{\left| |r| - \left| \frac{y}{2} + R \right| \right|}, \tag{4}$$

and

$$|b_3| \min = \frac{\left| b_g \frac{k}{2} \right|}{\left| |r| + \left| \frac{y}{2} + R \right| \right|}. \tag{5}$$

The average value of these two responses may be written

$$|b_3|_{av} = 1/2(|b_3|_{max} + |b_3|_{min}) = \left| b_g \frac{k}{2} \right| \frac{|r|}{\left| r^2 - \left| \frac{y}{2} + R \right|^2 \right|}. \tag{6}$$

For procedure 1), the AST is adjusted for zero reflection ($\Gamma_L = 0$), and the response may be written

$$|b_3|_0 = \left| b_g \frac{k}{2} \right| \frac{2}{|y + 2|}. \tag{7}$$

The ratio of these two responses is a measure of the directivity and the apparent directivity is given by

$$D_{A1} = 20 \log_{10} \frac{|b_3|_{av}}{|b_3|_0} = 20 \log_{10} \frac{|y+2|}{2} \left(\frac{|r|}{\left| r^2 - \left| \frac{y}{2} + R \right|^2 \right|} \right). \tag{8}$$

In procedure 2), it is convenient to express the response as

$$|b_3| = |b_g| \frac{|k|}{\left| y + \frac{2}{1 + K\Gamma_L e^{-2j\beta l}} \right|}, \tag{9}$$

[5] A similar technique is described by H. C. Poulter in "A note on measuring coaxial coupler directivity," *Hewlett-Packard J.*, vol. 8, pp. 1–4; May–June, 1957.

[6] A. C. MacPherson and D. M. Kerns, "A new technique for the measurement of microwave standing-wave ratios," Proc. IRE, vol. 44, pp 1024–1030; August, 1956.

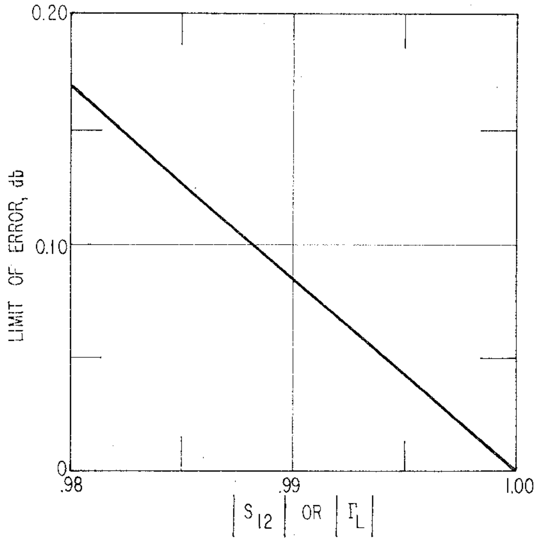

Fig. 3—Limit of error calculated from (17a) (Term 1) and (20).

where βl is the electrical length between reference planes of the load and the junction. One adjusts $|\Gamma_L|$ and l to give a null response and it is apparent from (9) that $K\Gamma_L e^{-2j\beta l} = -1$. In deriving the equation for maximum response as l is varied, one considers that the locus of

$$\frac{2}{1 + K\Gamma_L e^{-2j\beta l}},$$

when $|K\Gamma_L| = 1$, is a straight line parallel to the imaginary axis through the point (1, 0) in the complex plane. The maximum response may be written as

$$|b_3|_T = \frac{|b_g k|}{g + 1}, \qquad (10)$$

where g is the real part of y. Therefore, the apparent directivity may be written as

$$D_{A2} = 20 \log_{10} \frac{2|b_3|_{av}}{|b_3|_T}$$

$$= 20 \log_{10} (g + 1) \left\{ \frac{|r|}{\left| r^2 - \left| \frac{y}{2} + R \right|^2 \right|} \right\}. \qquad (11)$$

Error Analysis

The sources of error can be evaluated by considering the true directivity, defined by

$$D_T = 20 \log_{10} \frac{|S_{23}|}{|S_{13}|}, \qquad (12)$$

and evaluating the error in db as

$$\epsilon = D_T - D_A \qquad (13)$$

where D_A is given by (8) and (11) for procedures 1) and 2), respectively.

The following three conditions are sufficient to reduce the error to zero:

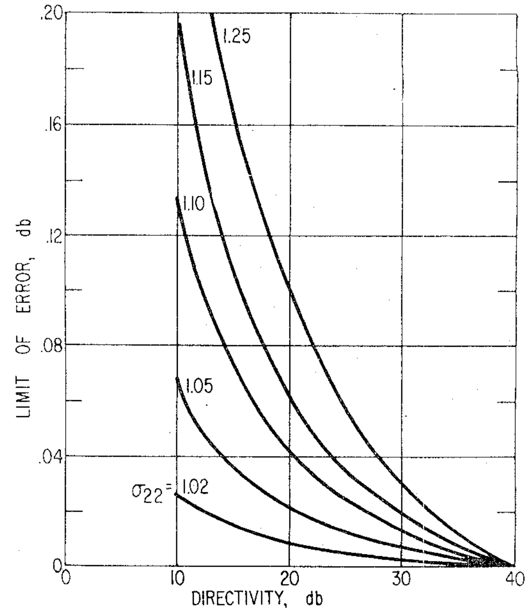

Fig. 4—Limit of error calculated from (17a) (Term 2).

$y = 0$, which reduces the ratios in (8) and (11) to $|K\Gamma_L|$; (14)

$|\Gamma_L| = 1$, which reduces (14) to $|K|$; (15)

$S_{22} = 0$, $S_{12} = 1$, which reduce (15) to $\dfrac{|S_{23}|}{|S_{13}|}$. (16)

It is assumed that departures from these conditions will be small and are therefore considered as individual cases.

Case I

$S_{22} \neq 0$ and $S_{12} \neq 1$, but other conditions are satisfied. Since D_{A1} and D_{A2} reduce to 20 $\log_{10} |K|$ in Case I, the error is

$$\epsilon_I = 20 \log_{10} \left| \frac{S_{23}}{S_{12}S_{23} - S_{13}S_{22}} \right|$$

$$= 20 \log_{10} \left| \frac{1}{S_{12}\left(1 - \frac{S_{13}S_{22}}{S_{12}S_{23}}\right)} \right|. \qquad (17)$$

One can separate (17) into two terms. If the coupling in decibels is 10 db or more, S_{12} may be eliminated from the second term with little loss of accuracy:

$$\epsilon_I \approx -20 \log_{10} |S_{12}| - 20 \log_{10} \left| 1 - \frac{S_{22}}{\frac{S_{23}}{S_{13}}} \right|. \qquad (17a)$$

The magnitude of the contribution from the first and second term is shown in Figs. 3 and 4, respectively. If the coupling in decibels is less than 10 db, the contribution of the second term is appreciably larger than shown in Fig. 4 and it is advisable to employ (17) for a more accurate estimate of the error. (In these cases where the error caused by assuming $|S_{12}| = 1$ would be large, it would probably be desirable to determine $|S_{12}|$ and make a correction to the measured ratio.)

Case II

$y \neq 0$, but other conditions are satisfied. Considerable manipulation of (12) minus (8) and use of the approximation

$$\frac{y}{2} \approx \frac{\Gamma_{2i}}{K}$$

yields for procedure 1),

$$20 \log_{10} \frac{1 - 2\left|\frac{\Gamma_{2i}}{K}\right| - 4|\Gamma_{2i}|^2}{1 + \left|\frac{\Gamma_{2i}}{K}\right|} \leq \epsilon_{II,1}$$

$$\leq 20 \log_{10} \frac{1 + 2\left|\frac{\Gamma_{2i}}{K}\right| - 4|\Gamma_{2i}|^2}{\left|\frac{\Gamma_{2i}}{K}\right|} \tag{18}$$

where Γ_{2i} is the reflection coefficient of arm 2, with the inactive generator and detector connected to arms 1 and 3. For the second procedure the limits of error after similar manipulation of (12) minus (11) may be expressed as

$$20 \log_{10} \frac{1 - 2\left|\frac{\Gamma_{2i}}{K}\right| - 4|\Gamma_{2i}|^2}{\left|\frac{\Gamma_{2i}}{K}\right|} \leq \epsilon_{II,2}$$

$$\leq 20 \log_{10} \frac{1 + 2\left|\frac{\Gamma_{2i}}{K}\right| - 4|\Gamma_{2i}|^2}{1 - 2\left|\frac{\Gamma_{2i}}{K}\right|}. \tag{19}$$

This expression differs from $\epsilon_{II,1}$ only in the denominator.

Fig. 5 shows the limits of error for both procedures 1) and 2) as a function σ_{2i}, (the VSWR corresponding to Γ_{2i}). The solid lines indicate the limits of error for procedure 1) and the dashed lines the limits of error for procedure 2). The limiting values for the error as the directivity becomes infinite are indicated on the right-hand side of the graph.

Case III

An imperfect short circuit, $|\Gamma_L| \neq 1$, but other conditions are satisfied.

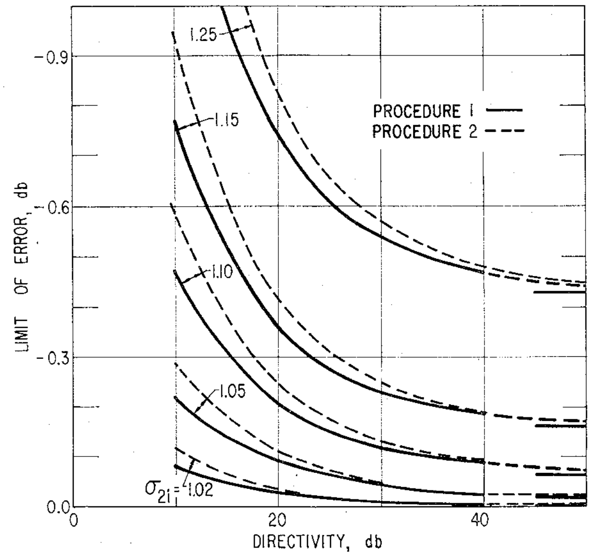

Fig. 5—Limit of error calculated from (18) and (19).

By use of (16) and implications of (17), the error may be written

$$\epsilon_{III} = -20 \log_{10} |\Gamma_L|. \tag{20}$$

This is of the same form as the first term of (17a) and is included in Fig. 3 as an alternate abscissa. This formula applies to both procedures.

The following example shows the limits of error for a case that might be considered typical. For a well-constructed, high directivity, 20-db coupler ($|S_{23}| = 0.1$), the absolute values of the other coefficients are usually

$$|S_{11}| \leq 0.025, \quad |S_{12}| \approx 0.995,$$
$$|S_{22}| \leq 0.025, \quad |S_{13}| < 0.001.$$
$$|S_{33}| \leq 0.025,$$

With the preliminary adjustment of $|\Gamma_{2i}|$ to less than 0.01, and use of a short circuit with a reflection coefficient of magnitude greater than 0.995, the limits of error from the above sources are -0.01 to $+0.08$. By measuring $|S_{12}|$ and $|\Gamma_L|$, and applying corrections for their departures from unity, the limits of error are reduced to -0.01 to 0.00 db. One must also consider the error made in measuring the ratio

$$\frac{|b_3|_{av}}{|b_3|_0} \quad \text{or} \quad \frac{|b_3|_{av}}{|b_3|_T}.$$

Using IF substitution techniques with a below-cutoff standard attenuator, the error may be held to ± 0.05 db with normal precautions. Unless this error is reduced to ± 0.01 db or less, the error in measuring directivity in this example would be limited by the error in measuring the above ratios.